Quantum Computing PHYS-541, Project 1

Teacher : vincenzo.savona@epfl.ch
Assistant : sara.alvesdossantos@epfl.ch, david.linteau@epfl.ch, shao.chiew@epfl.ch

Quantum supremacy analysis on a small quantum computer

This project is based on Google's theoretical article about how to assess the quantum advantage of a quantum computer. You can find the article here on arxiv.org. The idea was then used for the famous demonstration published in Nature in October 2019.

The idea is to use random quantum circuits (RQCs) and to sample the distribution of probabilities of the outputs, when the circuits are measured on the computational basis. It is argued that, for the statistical ensemble of RQCs of a given type, this probability distribution function has a universal shape, namely the Porter-Thomas distribution. This result originates from the theory of quantum chaos and disordered quantum systems. If we had an ideal quantum computer without errors, it would be enough to run RQCs several times and sample the output distribution and show that it corresponds to the expected one. The universality of the result is key, in that it allows us to know the expected output even for RQCs so big that they would be intractable with classical computers.

However NISQ quantum computers are characterized by errors. Then the probability distribution sampled at the output of the RQC will not correspond to the Porther-Thomas one. In the limit of very large error rate, the randomness induced by errors will dominate the RQC and the result will be a uniform probability distribution. We therefore know the result in the two limiting cases of no errors and many errors. For an intermediate case, one can show that the probability distribution has an intermediate shape between uniform and Porter-Thomas. The nontrivial aspect of the analysis by Google consists in determining where their quantum hardware lies in terms of error rates, and to estimate the *fidelity* of the RQC output with respect to the ideal one. Since the ideal circuit cannot be simulated, the google collaboration has found a very smart way of extrapolating the fidelity of their quantum computation, with a technique called cross-entropy benchmark. It turns out that the fidelity of their quantum computation is $\mathcal{F} = 0.3\%$. Even with such a low fidelity, the result by the quantum computer would be extremely hard to simulate on a classical hardware.

The goal of the project is:

1. Read and understand the main article and present its results.
2. Implement a RQC model on a IBM-Q machine (e.g. the 15-qubit one).
3. Repeat the cross-entropy benchmark analysis for this machine, or for the corresponding QASM simulation including simulated errors.
4. Determine the fidelity of the machine as a function of the depth of the RQC
5. (Optional). Search the literature for proofs that the RQC model under investigation is classically intractable even in presence of an arbitrary error rate.

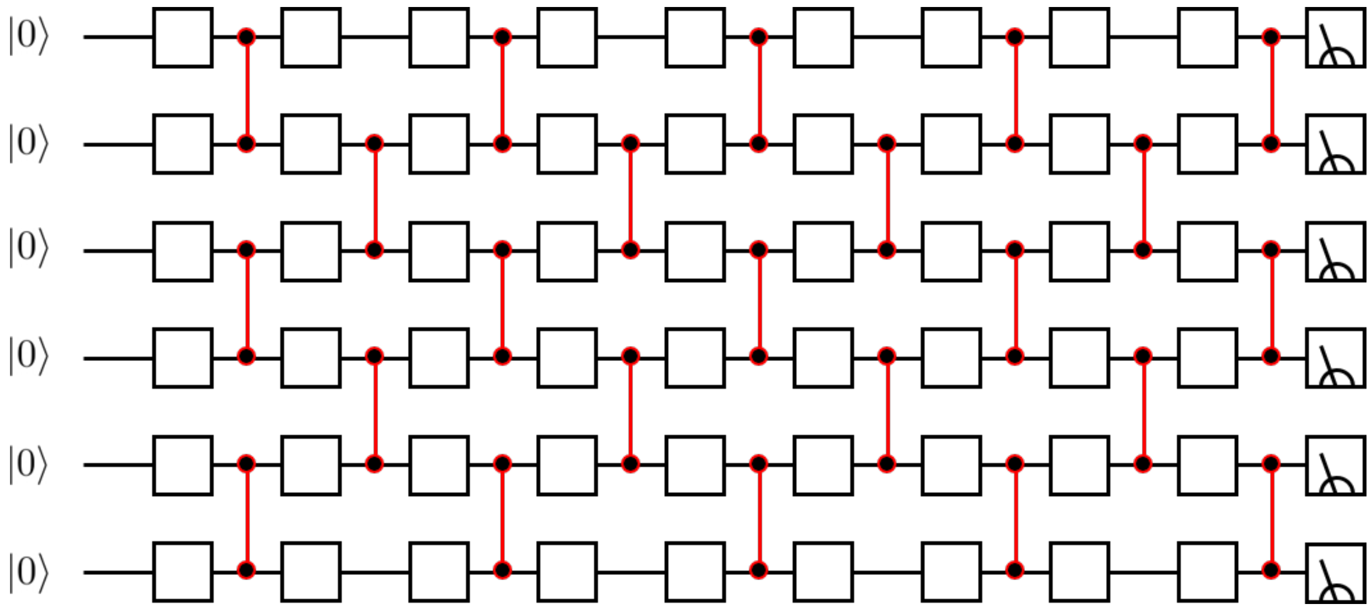

Figure 1: A sketch of the RQC we are going to consider. Red gates are controlled-Z gates. Empty squares are single-qubit gates chosen randomly among the three gates listed below. An important prescription is not to have twice the same one-qubit gate on the same qubit in neighboring layers. In this sketch there are $n = 6$ qubits and $m = 9$ cycles.

As a RQC model, for simplicity we will use a one-dimensional model. This means that qubits may be pictured as being arranged in a one-dimensional chain, in such a way that two-qubit gates can only be applied between one qubit and one of its two neighboring qubits. More precisely we will adopt the brickwork RQC model depicted in the figure. There, the two qubit gates are controlled-Z gates, and the single qubit gates are chosen randomly from a set of the following three gates.

$$X^{1/2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$
$$Y^{1/2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$
$$W^{1/2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\sqrt{i} \\ \sqrt{-i} & 1 \end{pmatrix}$$

The circuit alternates a layer of single-qubit gates to a layer of controlled-Z gates. Each pair of layers is called a cycle, and the depth of the circuit is measured in terms of the number of cycles m, while its width is given by the number of qubits n.